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Using Statistics to Understand Extreme Values with Application to Hydrology

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Abstract– Environmental issues are extremely important to human existence. These issues vary from various pollution levels to climate change. They bring hazardous impacts to man in both developed and developing countries most especially when extreme cases are experienced. These extreme cases are more dangerous in developing countries than in developed countries due to inadequate monitoring research and projection. It is therefore necessary that we quantify these extreme changes in environmental problems statistically; these efforts will be tailored towards achieving sustainability using the appropriate statistical methodologies.
This research work studies the applications of stochastic models for understanding extreme values in hydrology using monthly rainfall data from 1956-2013 in the south-western zone, Nigeria. The exploratory data analysis tools used reveal the presence of extreme values in the data. These may indicate the need to study the data. The Block Maxima Method was used to select the extreme values and three types of extreme value distributions (Type I, Type II and Type III) were used, the 3 parameters case of Type II and Type III extreme value distributions were studied and compared. The different return periods and return levels were found and it was discovered that the return levels increased over the periods (years). The implication of this result is that there is tendency of extremely high rainfall in the nearest future. The attention of governments and stakeholders must be drawn to this case as it may lead to continuous flooding as currently being experienced.

Index Terms– Hydrology, Pollution, Extreme Value Distributions, Return Levels, Flooding, Parameters, Stochastic.

1 INTRODUCTION

Stochastic models are very useful in understanding the behaviour of different phenomena because events such as floods, hurricanes, earthquakes, stock market crashes and so on are natural phenomena which seem to follow no rule however, researches have helped to discover distributions that can readily model these extreme events (Chavez and Roehrl, 2004). The last two decades in many cities in the world have been associated with extreme events and therefore there is a need to assess the probability of these rare events. Tella (2011); the weather has been funny; seasons have been shifting fairly unpredictably. In today's world, we are having shorter winter and longer summer in the temperate region. In the tropics, we are experiencing shorter rainy season, longer dry season and shorter harmattan or cold wind. Rains are coming late and they come furiously when they fall, ocean levels are rising and breaking their boundaries causing landslide and massive flooding (Pakistan flood disaster 2010; Ibadan flood disaster 2011; hurricane katrina that ravaged the gulf states of USA (New Orleans) 2005; Southern California wildfire 2009 which forced 500 000 residents to flee their homes; Australian fire disaster 2008 and 2009; Russia fire disaster 2010; Markudi flood disaster 2012; Lagos ocean surge i.e. Kuramo beach 2012; alarming warning of ocean surge in River Niger by National Emergency Management Authority 2012; Nepal earthquake 2015 e.t.c.).

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Countries in semi-arid regions are experiencing less rainfall and more droughts. In 2008, Darfur in Sudan experienced severe drought causing a huge loss of the human population. Tella (2011) pointed out that Lesotho experienced high temperature and drought which destroyed crops and caused a huge loss of the human population.

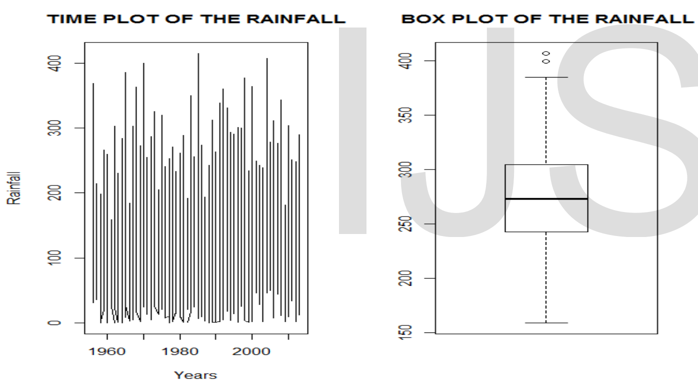
2 THE EXTREME VALUE ANALYSIS

Extreme value distributions occur as limiting distributions for maximum or minimum (extreme values) of a sample of independent and identically distributed random variables as the sample size increases. Extreme Value Theory (EVT) or Extreme Value Analysis (EVA) is a branch of Statistics dealing with the extreme deviations from the median of probability distributions from a given ordered sample of a given random variable, the probability of events that are more extreme than any observed prior samples. It is a discipline that develops statistical techniques for describing the unusual phenomena such as floods, wind guests, air pollution, earthquakes, risk management, insurance and financial matters (Lakshminarayan, 2006). Extreme Value Theory and Distribution has found applications in hydrology, engineering, environmental research and meteorology, stock market and finance. The prediction of earthquake magnitudes (1755 Lisbon earthquakes), modeling of extremely high temperatures and rainfalls, and the prediction of high return levels of wind speed relevant for the design of civil engineering structure have been carried out using extreme value distribution (Gumbel).

A review of this statistical methodology can be found in Broussard and Booth (1998), Behr *et al.*, (1991), Xapson *et al.*,(1998), Lee (1992), Yasuda and Mori (1997), Jan *et al.*,(2004), Jandhyala *et al.*,(1999), Kartz and Brown (1992), Charles (1995), Richard *et al.,* (2002), Hosking (1985,1990), Manfred and Evis (2006), Koutsoyiannis (2004; 2009), Hosking *et al.,* (1985), Isabel and Claudia (2010), and Olivia and Jonathan (2012). Alexander and McNeil (1999) used Extreme Value Theory (EVT) in risk management (RM) as a method for modelling and measuring extreme risks; considered Peak Over Threshold (POT) model and emphasized the generality of the approach. Within the POT class of models, there are two styles of analysis namely: semi-parametric models- built around the Hill estimator and its relatives (Beirlant *et al.*,1996, Daneilsson *et al.,* 1998); and the fully parametric models- based on the Generalized Pareto Distribution i.e. GPD (Embrechts *et al.*,1998). Alexander (1999) used GPD as probability distribution for risk management and considered it as equally important as (if not more important than) the Normal distribution because Normal distribution cannot model certain market returns with infinite fourth moment.

3 MATERIALS AND METHODS

3.1 Exploratory Data Analysis



Figs. 1&2: Time and Box Plot of the Rainfall

The time plot shows the systematic movement of the series or data over a period of time, it shows that there are extreme values i.e. maximum values, and the box plot confirms the presence of extreme values which calls for urgent attention because they are rare events and hazardous.

3.2 Estimation of Parameters of Extreme Value Distributions

There are different methods of estimating the parameters of the extreme value distributions namely: Maximum Likelihood (ML) estimations in the possible presence of covariates, Probability Weighted Moment (PWM) or (L- Moments), Least Square Method e.t.c. Probablity Weighted Moments (PWM or L- Moments) are more popular than ML in applications to hydrologic extreme because of their computational simplicity and their good technique performance for small samples (Hosking 1985, 1990). Though Probability Weighted Moments technique has the disadvantage of not being able to readily incorporate covariates. On the other hand, it is better to apply Maximum Likelihood technique in the presence of covariates (Richard *et al.,* 2002). One advantage of the Maximum Likelihood Method is that approximate standard errors for estimated parameters and design values can be automatically produced either through the information matrix e.g. extremes software (Farago and Katz, 1990) or through profile likelihood (Coles, 2001). But like the parameter estimates themselves, such standard errors can be quite unreliable for small sample sizes. The Maximum Likelihood technique would be applied in the research because the sample size is large (n > 25) and the technique produces minimum variance of the estimated parameters i.e. produces reliable approximate standard errors for estimated parameters.

3.3 Maximum Likelihood Estimation (MLE) of Gumbel Distribution

The MLE (Hanter and More 1965) is the parameter that maximizes the log of likelihood function L. Given independent and identically distributed random variables X, the likelihood function L is given as:

$$L = f(x_1,\theta).f(x_2,\theta)\ldots f(x_n,\theta) = L = \prod_{i=1}^{n} f(x_i,\theta)$$

Given Gumbel distribution for maximum case with $f(x) =$

$$\frac{1}{\sigma}\exp\left[-\left(\frac{x-\mu}{\sigma}\right) - exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)\right] \quad x \in \text{IN or Z} \tag{1}$$

$$L = \frac{1}{\sigma}exp\left[-\left(\frac{x_1-\mu}{\sigma}\right) - exp\left(-\left(\frac{x_1-\mu}{\sigma}\right)\right)\right] \times \frac{1}{\sigma}exp\left[-\left(\frac{x_2-\mu}{\sigma}\right) - exp\left(-\left(\frac{x_2-\mu}{\sigma}\right)\right)\right] \ldots \frac{1}{\sigma}exp\left[-\left(\frac{x_n-\mu}{\sigma}\right) exp\left(-\left(\frac{x_n-\mu}{\sigma}\right)\right)\right] \tag{2}$$

$$L(\mu,\sigma) = f(x_1 \ldots, x_n/\mu,\sigma) = \prod_{i=1}^{n} f\ (x_i/\mu,\sigma)$$

$$L = \prod_{i=1}^{n}\frac{1}{\sigma}\,exp\left[-\left(\frac{x_i-\mu}{\sigma}\right) - exp\left(-\left(\frac{x_i-\mu}{\sigma}\right)\right)\right] \tag{3}$$

$$= \left(\frac{1}{\sigma}\right)^n \exp\left[-\sum_{i=1}^{n} x_i\left(\frac{x_i-\mu}{\sigma}\right) + exp\left(-\left(\frac{x_i-\mu}{\sigma}\right)\right)\right] \tag{4}$$

$$l = \ln(L) = -n\ln(\sigma) - \sum_{i=1}^{n}\left[\left(\frac{x_i-\mu}{\sigma}\right) + exp\left(-\left(\frac{x_i-\mu}{\sigma}\right)\right)\right] \tag{5}$$

$$\frac{\partial l}{\partial \mu} = -\sum_{i=1}^{n}\left[-\frac{1}{\sigma} + \frac{1}{\sigma}exp\left(\frac{x_i-\mu}{\sigma}\right)\right] = \frac{n}{\sigma} - \frac{1}{\sigma}\sum_{i=1}^{n} exp\left[-\frac{(x_i-\mu)}{\sigma}\right] \tag{6}$$

$$\frac{\partial l}{\partial \sigma} = -\frac{n}{\sigma} - \sum_{i=1}^{n} \left[-\left(\frac{x_i-\mu}{\sigma^2}\right) + \left(\frac{x_i-\mu}{\sigma^2}\right) exp\left[\left(\frac{x_i-\mu}{\sigma}\right)\right]\right] = -\frac{n}{\sigma} + \sum_{i=1}^{n} \left(\frac{x_i-\mu}{\sigma^2}\right) - \sum_{i=1}^{n} \left(\frac{x_i-\mu}{\sigma^2}\right) exp\left[-\left(\frac{x_i-\mu}{\sigma^2}\right)\right] \quad (7)$$

By maximization, we choose μ, σ such that

$$\begin{bmatrix} \frac{\partial l}{\partial \mu} \\ \frac{\partial l}{\partial \sigma} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using Newton – Raphson Algorithm (Second order)

$$\Rightarrow \frac{\partial^2 l}{\partial \mu^2} = -\frac{1}{\sigma}\sum_{i=1}^{n} \left(\frac{1}{\sigma}\right) exp\left[-\left(\frac{x_i-\mu}{\sigma}\right)\right] = -\frac{1}{\sigma^2}\sum_{i=1}^{n} exp\left[-\left(\frac{x_i-\mu}{\sigma}\right)\right] \quad (8)$$

$$\frac{\partial^2 l}{\partial \mu \partial \sigma} = -\frac{n}{\sigma^2} + \frac{1}{\sigma^2}\sum_{i=1}^{n} exp\left[-\left(\frac{x_i-\mu}{\sigma}\right)\right] - \frac{1}{\sigma}\sum_{i=1}^{n} \left(\frac{x_i-\mu}{\sigma^2}\right) exp\left[-\frac{(x_i-\mu)}{\sigma}\right] \quad (9)$$

$$\frac{\partial^2 l}{\partial \mu \partial \sigma} = -\frac{n}{\sigma^2} + \frac{1}{\sigma^2}\sum_{i=1}^{n} exp\left[-\frac{(x_i-\mu)}{\sigma}\right] - \frac{1}{\sigma^3}\sum_{i=1}^{n} (x_i-\mu) exp\left[\frac{-(x_i-\mu)}{\sigma}\right] \quad (10)$$

$$\frac{\partial^2 l}{\partial \sigma^2} = \frac{n}{\sigma^2} - \frac{2}{\sigma^3}\sum_{i=1}^{n} (x_i-\mu) + \frac{2}{\sigma^3}\sum_{i=1}^{n} (x_i-\mu) exp\left[-\frac{(x_i-\mu)}{\sigma}\right] - \sum_{i=1}^{n} \frac{(x_i-\mu)}{\sigma^2}\frac{(x_i-\mu)}{\sigma^2} exp\left[-\frac{(x_i-\mu)}{\sigma}\right] \quad (11)$$

$$= \frac{n}{\sigma^2} - \frac{2}{\sigma^3}\sum_{i=1}^{n} (x_i-\mu) + \frac{2}{\sigma^3}\sum_{i=1}^{n} (x_i-\mu) exp\left[-\frac{(x_i-\mu)}{\sigma}\right] - \frac{1}{\sigma^4}\sum_{i=1}^{n} (x_i-\mu)^2\, exp\left[-\frac{(x_i-\mu)}{\sigma}\right] \quad (12)$$

$$H = \begin{bmatrix} \frac{\partial^2 l}{\partial \mu^2} & \frac{\partial^2 l}{\partial \mu \partial \sigma} \\ \frac{\partial^2 l}{\partial \mu \partial \sigma} & \frac{\partial^2 l}{\partial \sigma^2} \end{bmatrix}$$

Where H is called Hessian matrix

Also, $g = \begin{bmatrix} \frac{\partial l}{\partial \mu} \\ \frac{\partial l}{\partial \sigma} \end{bmatrix}$

∴ To obtain estimates of μ, σ

$$\begin{bmatrix} \mu^{(1)} \\ \sigma^{(1)} \end{bmatrix} = \begin{bmatrix} \mu^{(0)} \\ \sigma^{(0)} \end{bmatrix} - H^{-1}\left(\mu^{(0)}\, \sigma^{(0)}\right) g\left(\mu^{(0)}\, \sigma^{(0)}\right)$$

The iteration converges when

$$\left[\begin{bmatrix} \mu^{(1)} \\ \sigma^{(1)} \end{bmatrix} - \begin{bmatrix} \mu^{(0)} \\ \sigma^{(0)} \end{bmatrix}\right]^1 \left[\begin{bmatrix} \mu^{(1)} \\ \sigma^{(1)} \end{bmatrix} - \begin{bmatrix} \mu^{(0)} \\ \sigma^{(0)} \end{bmatrix}\right] \leq \text{k or}$$

$$\left(\mu^{(1)} - \mu^{(0)}\right)^2 + \left(\sigma^{(1)} - \sigma^{(0)}\right)^2 < \text{k}$$

Where $\alpha^{(0)}, \mu^{(0)}$ and $\sigma^{(0)}$ are initial values; $\mu^{(1)}$ and $\sigma^{(1)}$ are iterative values, and k is the tolerance level for the change. The final value of $\mu^{(1)}$ and $\sigma^{(1)}$ are the estimates after the convergence of the iteration.

3.4 Maximum Likelihood Estimation of 3 Parameters Weibull Distribution

Given a random samples $x_1, x_2, \ldots, x_n$, the likelihood function of 3-parameter weibull distribution is given as $L(\mu, \sigma, \alpha)$ =

$$\alpha^n \sigma^{-n\alpha} \left[\prod_{i=1}^{n} (x_i-\mu)\right]^{\alpha-1} \exp\left[-\sigma^{-\alpha}\sum_{i=1}^{n} (x_i-\mu)^{\alpha}\right]$$

By maximizing the log of likelihood functions to obtain estimate of μ, σ and α i.e.

$\frac{\partial \log L}{\partial \mu} = 0, \frac{\partial \log L}{\partial \sigma} = 0$ and $\frac{\partial \log L}{\partial \alpha} = 0$

This generates three equations namely

$$\frac{n}{\alpha} + \sum_{i=1}^{n} \log(x_i-\mu) = \frac{n\sum_{i=1}^{n} (x_i-\mu)^{\alpha} \log(x_i-\mu)}{\sum_{i=1}^{n} (x_i-\mu)^{\alpha}} \quad (13)$$

$$\frac{n\alpha \sum_{i=1}^{n} (x_i-\mu)^{\alpha-1}}{\sum_{i=1}^{n} (x_i-\mu)^{\alpha}} = (\alpha-1)\sum_{i=1}^{n} \left(\frac{1}{x_i-\mu}\right) \quad (14)$$

$$\text{and } \hat{\sigma} = \left[\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^{\alpha}\right]^{\frac{1}{\alpha}} \quad (15)$$

The equations can be solved numerically to obtain $\hat{\mu}, \hat{\sigma}$ and $\hat{\alpha}$. The other form of three parameters weibull distribution is given as:

$$f(x) = \frac{\alpha}{v-\mu}\left(\frac{x-\mu}{v-\mu}\right)^{\alpha-1} exp\left[-\left(\frac{x-\mu}{v-\mu}\right)^{\alpha}\right] \quad (16)$$

Where $\sigma = v - \mu$ and v is called characteristic value.

The Likelihood function L is given as

$$L = \prod_{i=1}^{n} f(x_i) = \left(\frac{\alpha}{v-\mu}\right)^{n} \prod_{i=1}^{n}\left(\frac{x_i-\mu}{v-\mu}\right)^{\alpha-1} exp\left[-\left(\frac{x-\mu}{v-\mu}\right)^{\alpha}\right] \quad (17)$$

$$\log(L) = n\ln\left(\frac{\alpha}{v-\mu}\right) + \sum_{i=1}^{n}\ln\left(\frac{x_i-\mu}{v-\mu}\right)^{\alpha-1} - \sum_{i=1}^{n}\left(\frac{x_i-\mu}{v-\mu}\right)^{\alpha} \quad (18)$$

$$= n\ln\alpha - n\alpha\ln(v-\mu) + (\alpha-1)\sum_{i=1}^{n}\ln(x_i-\mu) - \sum_{i=1}^{n}\left(\frac{x_i-\mu}{v-\mu}\right)^{\alpha} \quad (19)$$

Let $w = v - \mu$ to reduce the magnitude of the number in equation (19)

$$\Rightarrow \log(L) = n\ln\alpha - n\alpha\ln w + (\alpha-1)\sum_{i=1}^{n}\ln w - \sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha} \quad (20)$$

$$\Rightarrow L^* = \frac{\log(L)}{n} = \ln\alpha - \alpha\ln w + (\alpha-1)\frac{1}{n}\sum_{i=1}^{n}\ln(x_i-\mu) - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha} \quad (21)$$

$$\frac{\partial L^*}{\partial w} = \frac{-\alpha}{w} + \frac{\alpha}{w}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha} = 0 \quad (22)$$

$$\frac{\partial L^*}{\partial \alpha} = \frac{1}{\alpha} - \ln w + \frac{1}{n}\sum_{i=1}^{n}\ln(x_i-\mu) - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha}\ln(x_i-\mu) + \ln w\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha} = 0 \quad (23)$$

$$\frac{\partial L^*}{\partial \mu} = \frac{\alpha}{w}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha-1} - (\alpha-1)\frac{1}{n}\sum_{i=1}^{n}\ln(x_i-\mu)^{-1} = 0 \quad (24)$$

Then μ , α and w can be obtained using second order Newton – Raphson method. It requires to obtain Hessian matrix H given by:

$$H = \begin{bmatrix} \frac{\partial^2 L^*}{\partial w^2} & \frac{\partial^2 L^*}{\partial w \partial \alpha} & \frac{\partial^2 L^*}{\partial w \partial \mu} \\ \frac{\partial^2 L^*}{\partial \alpha \partial w} & \frac{\partial^2 L^*}{\partial \alpha^2} & \frac{\partial^2 L^*}{\partial \mu \partial \alpha} \\ \frac{\partial^2 L^*}{\partial \mu \partial w} & \frac{\partial^2 L^*}{\partial \mu \partial \alpha} & \frac{\partial^2 L^*}{\partial \mu^2} \end{bmatrix}$$, Where

$$\frac{\partial^2 L^*}{\partial w^2} = \frac{\alpha}{w^2} - \frac{\alpha(\alpha+1)}{w^2}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha} \quad (25)$$

$$\frac{\partial^2 L^*}{\partial \alpha^2} = -\frac{1}{\alpha^2} - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha}\left(ln(x_i-\mu)\right)^2 - (lnw)^2$$

$$\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha} + 2\ln w\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)\ln(x_i-\mu) \quad (26)$$

$$\frac{\partial^2 L^*}{\partial \mu^2} = -\alpha\frac{(\alpha-1)}{w^2}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha-2} - (\alpha-1)\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{-2} \quad (27)$$

$$\frac{\partial^2 L^*}{\partial \alpha \partial w} = -\frac{1}{w} + \frac{\alpha}{w}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha}\ln(x_i-\mu) + \frac{1-\alpha}{w}$$

$$(\ln w)\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha} \quad (28)$$

$$\frac{\partial^2 L^*}{\partial \alpha \partial \mu} = -\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^{-1} + \frac{\alpha}{w}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha-1}\ln(x_i-\mu)$$

$$+\frac{(1-\alpha)}{w}\ln w\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha-1} \quad (29)$$

$$\frac{\partial^2 L^*}{\partial \mu \partial w} = \frac{-\alpha^2}{w^2}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{w}\right)^{\alpha-1} \quad (30)$$

and $f = \begin{bmatrix} \frac{\partial L^*}{\partial w} \\ \frac{\partial L^*}{\partial \alpha} \\ \frac{\partial L^*}{\partial \mu} \end{bmatrix}$

$$\begin{bmatrix} w^{(new)} \\ \alpha^{(new)} \\ \mu^{(new)} \end{bmatrix} = \begin{bmatrix} w^{(old)} \\ \alpha^{(old)} \\ \mu^{(old)} \end{bmatrix} - H^{-1}\left(w^{(old)}\ \alpha^{(old)}\ \mu^{(old)}\right) f\left(w^{(old)}\ \alpha^{(old)}\ \mu^{(old)}\right)$$

Note: The three parameters weibull distribution reduces to two parameters weibull distribution by setting equations (22), (23) and (24) to zero, then solve with i=j=2. The iteration converges if;

$$\left[\begin{bmatrix} w^{(new)} \\ \alpha^{(new)} \\ \mu^{(new)} \end{bmatrix} - \begin{bmatrix} w^{(old)} \\ \alpha^{(old)} \\ \mu^{(old)} \end{bmatrix}\right]^{1}\left[\begin{bmatrix} w^{(new)} \\ \alpha^{(new)} \\ \mu^{(new)} \end{bmatrix} - \begin{bmatrix} w^{(old)} \\ \alpha^{(old)} \\ \mu^{(old)} \end{bmatrix}\right] \le k$$

Where k is the level of tolerance.

3.5 Maximum Likelihood Estimation of 2 Parameters Weibull Distribution

$f(x) = \frac{\alpha}{\sigma}\left(\frac{x}{\sigma}\right)^{\alpha-1} exp\left[-\left(\frac{x}{\sigma}\right)^{\alpha}\right] \qquad \alpha > 0, \sigma > 0,$

$\mathrm{L}(x_1 \dots . x_n, \alpha, \sigma) = f(x_1). f(x_2), \dots, f(x_n)$

$$= \frac{\alpha}{\sigma}\left(\frac{x_1}{\sigma}\right)^{\alpha-1} exp\left[-\left(\frac{x_1}{\sigma}\right)^{\alpha}\right] \times \frac{\alpha}{\sigma}\left(\frac{x_2}{\sigma}\right)^{\alpha-1} exp\left[-\left(\frac{x_2}{\sigma}\right)^{\alpha}\right] \times \dots \times \frac{\alpha}{\sigma}\left(\frac{x_n}{\sigma}\right)^{\alpha-1} exp\left[-\left(\frac{x_n}{\sigma}\right)^{\alpha}\right] \quad (31)$$

$$\mathrm{L}(x_1, x_2, \dots x_n\ \alpha, \sigma) = \prod_{i=1}^{n}\left(\frac{\alpha}{\sigma}\right)\left[\left(\frac{x_i}{\sigma}\right)^{\alpha-1}\right] exp\left(\frac{-x_i}{\sigma}\right)^{\alpha} \quad (32)$$

$$\Rightarrow \frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \ln\ x_i - \frac{1}{\sigma}\sum_{i=1}^{n} x_i^{\alpha} \ln\ x_i = 0 \quad (33)$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^2}\sum_{i=1}^{n} x_i^{\alpha} = 0 \quad (34)$$

Eliminate σ in the equations (33) and (34), we have:

$$\frac{\sum_{i=1}^{n} x_i^{\alpha} \ln x_i}{\sum_{i=1}^{n} x_i^{\alpha}} - \frac{1}{\alpha} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i = 0 \quad (35)$$

α is obtained by using Newton–Raphson method i.e.

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)} \quad (36)$$

$$\text{where} \quad f(\alpha) = \frac{\sum_{i=1}^{n} x_i^{\alpha} \ln x_i}{\sum_{i=1}^{n} x_i^{\alpha}} - \frac{1}{\alpha} - \frac{1}{\mathrm{n}}\sum_{i=1}^{n} \ln x_i \quad (37)$$

$$f^1(\alpha) = \sum_{i=1}^{n} x_i^{\alpha}(\ln x_i)^2 - \frac{1}{\alpha^2}\sum_{i=1}^{n} x_i^{\alpha}(\alpha \ln x_i - 1) - \left(\frac{1}{\mathrm{n}}\sum_{i=1}^{n} \ln x_i\right)\left(\sum_{i=1}^{n} x_i^{\alpha} \ln x_i\right) \quad (38)$$

Once α is determined, σ can be estimated as $\sigma = \frac{\sum_{i=1}^{n} x_i^{\alpha}}{\mathrm{n}}$

3.6 Maximum Likelihood Estimation of Frechet Distribution

$$\mathrm{L}\ (\mu, \sigma, \alpha) = \alpha^{\mathrm{n}} \sigma^{\mathrm{n}\alpha}\left[\prod_{i=1}^{n}(x_i - \mu)\right]^{-\alpha+1} exp\left[-\sigma^{\alpha}\sum_{i=1}^{n}(x_i - \mu)^{-\alpha}\right]$$

$$\frac{\partial \mathrm{logL}}{\partial \mu} = 0, \frac{\partial \mathrm{logL}}{\partial \sigma} = 0\ and\ \frac{\partial \mathrm{logL}}{\partial \alpha} = 0$$

The derivatives give three unknown equations:

$$\frac{\mathrm{n}}{\hat{\alpha}} + \frac{\mathrm{n}\sum_{i=1}^{n}(x_i - \mu)^{-\alpha} \log(x_i - \mu)}{\sum_{i=1}^{n}(x_i - \mu)^{-\alpha}} = \sum_{i=1}^{n} \log(x_i - \mu) \quad (39)$$

$$\mathrm{n}\,\alpha \frac{\sum_{i=1}^{n}(x_i - \mu)^{-(\alpha+1)}}{\sum_{i=1}^{n}(x_i - \mu)^{-\alpha}} = (\hat{\alpha} + 1)\sum_{i=1}^{n}\frac{1}{(x_i - \mu)} \quad (40)$$

$$\text{and}\ \hat{\sigma} = \left[\frac{1}{\mathrm{n}}\sum_{n}^{1}(x_i - \mu)^{-\alpha}\right]^{-\frac{1}{\alpha}} \quad (41)$$

4 RESULTS AND DISCUSSION

4.1 Results

4.1.1 Model Diagnosis of Extreme Value Distributions

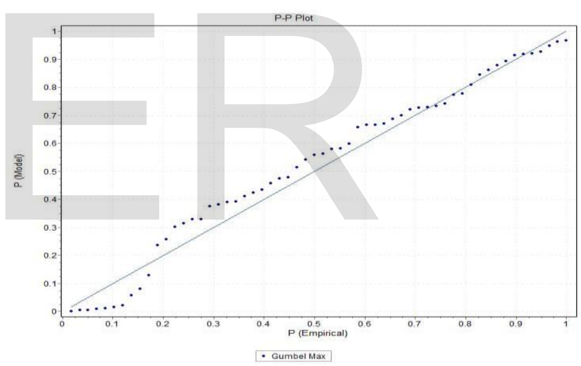


Fig. 3: Probability Plot of Gumbel Distribution

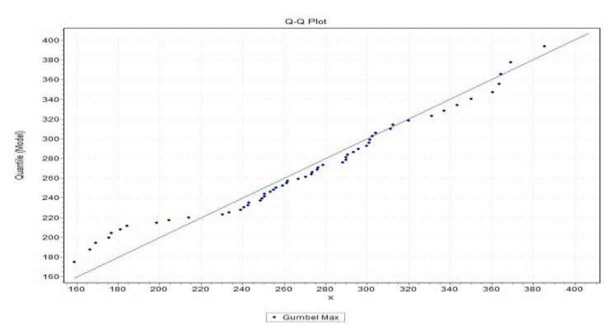


Fig. 4: Quantile – Quantile Plot of Gumbel Distribution

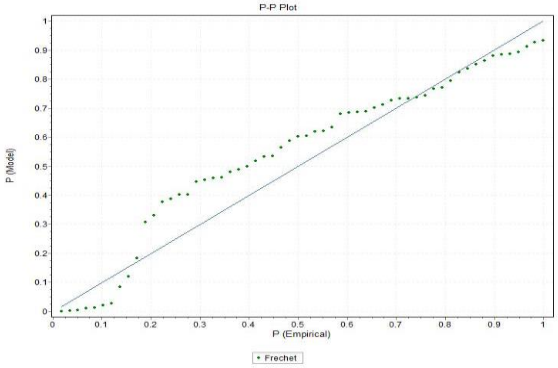


Fig. 5: Probability Plot of 2 Parameters Frechet Distribution

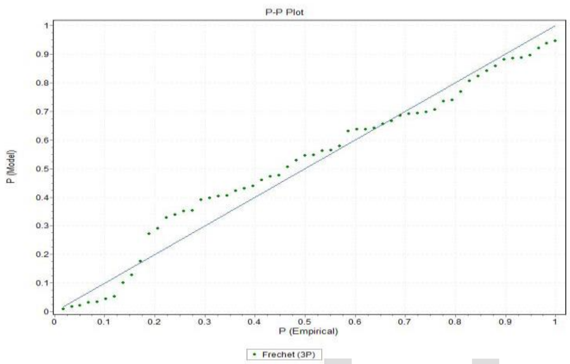


Fig. 6: Probability Plot of 3 Parameters Frechet Distribution

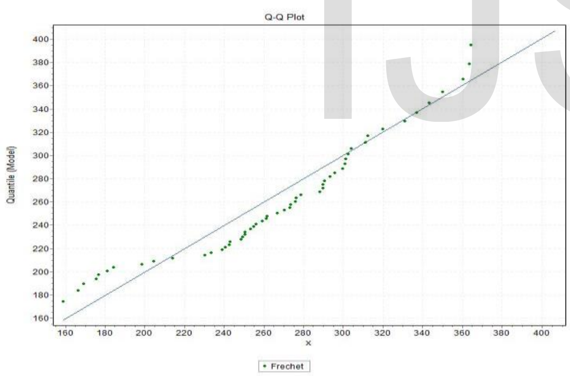


Fig. 7: Quantitle – Quantile Plot of 2 Parameters Frechet Distribution

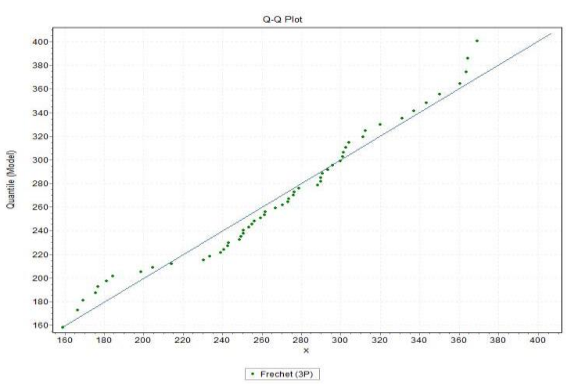


Fig. 8: Quantitle – Quantile Plot of 3 Parameters Frechet Distribution

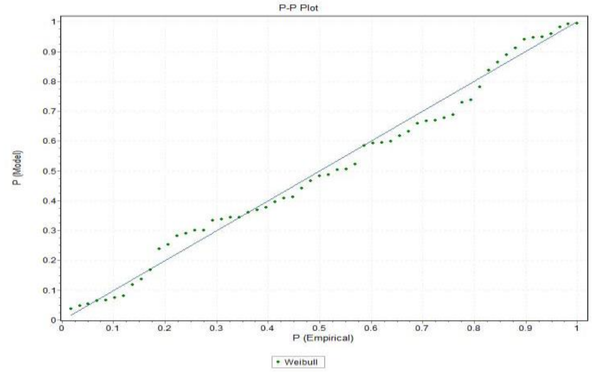


Fig. 9: Probability Plot of 2 Parameters Weibull Distribution

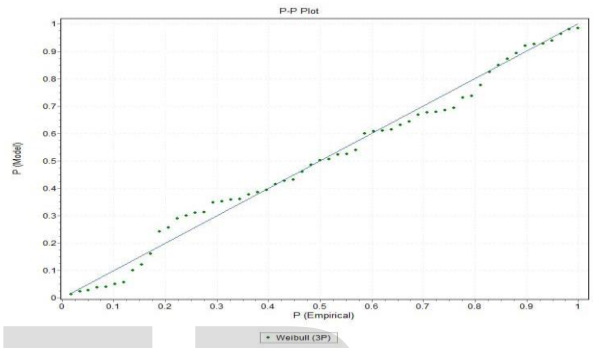


Fig. 10: Probability Plot of 3 Parameters Weibull Distribution

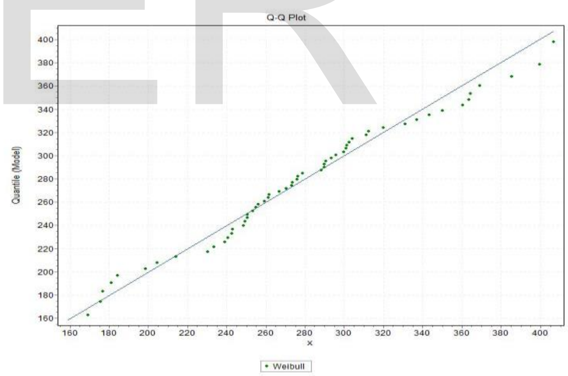


Fig. 11: Quantile-Quantile Plot of 2 Parameters Weibull Distribution

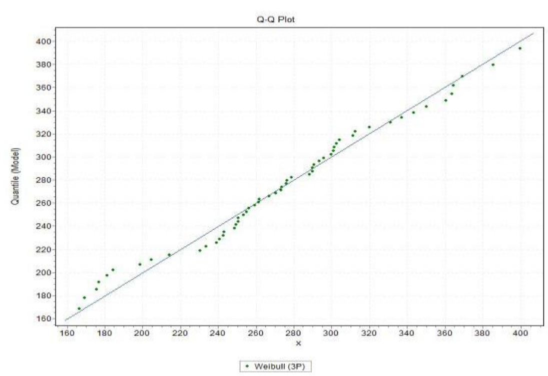


Fig.12: Quantile-Quantile Plot of 3 Parameters Weibull Distribution

4.1.2 Predicting the Probability of Exceedness using Gumbel Distribution

The probability that the maximum rainfall denoted by x_i will exceed this level (value) i.e. 407.1 is given as:

$$1-\exp\left[-\exp\left(-\frac{407.1-247.78}{46.705}\right)\right]$$

$P(x_i > 407.1) = 0.032\,4630\,29$

4.1.3 Predicting Return Period Using Gumbel Distribution

The return period of the Gumbel distribution is given as: $T = [P(x_i > k)]^{-1}$

$$\Rightarrow T = \left[P\left(x > 407.1\right)\right]^{-1}$$

$$= (0.032\,463\,029)^{-1} = 30.80 \approx 31 \text{ years}$$

4.1.4 Computing Return Levels for Different Return Periods

Assuming 100 years return period i.e. T= 100 years

$$P\left(x_i > U(T)\right) = \frac{1}{T}$$

$$\Rightarrow 1 - F\left(U(T)\right) = \frac{1}{T}$$

$$U(T) = F^{-1}\left(1 - \frac{1}{T}\right)$$

$$= F^{-1}(0.99) \text{ i.e. } U(100) = F^{-1}(0.99) = 426.6$$

The return levels for different return periods are given graphically as thus:

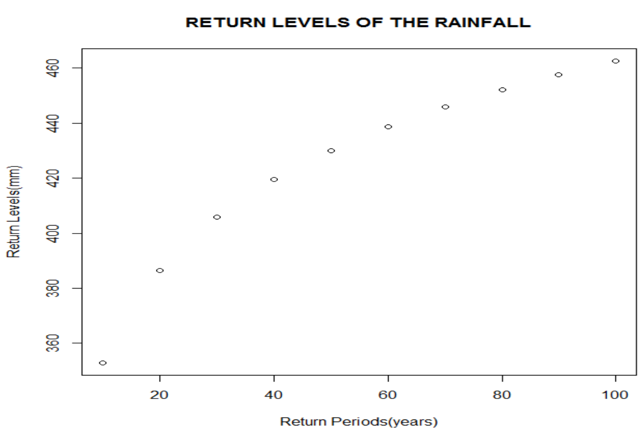


Fig. 13: Return Levels of the Rainfall

4.2 Discussion

This work examined the statistical model of extreme values in Hydrology. Exploratory Data Analysis (EDA) using various EDA Tools were carried out on the data and the Box Plot revealed on the whole data that there is the presence of extreme values; this demands urgent attention as this would have resulted in torrential rain which can lead to flood. The Block Maximum Method (BMM) was used to select the extreme values and the Maximum Likelihood Estimation method was used to estimate the parameters of the distributions, The extreme value distributions namely: Type I (Gumbel), Type II (Frechet) for both 2 parameters and 3 parameters cases and Type III (Weibull) for both 2 parameters and 3 parameters cases were used to model annual maximum rainfall of south western zone in Nigeria from 1956 – 2013. It was discovered that the 3 parameters of Type II and Type III distributions were more appropriate than 2 parameters as this is also evident in the work of Koutsoyiannis (Koutsoyiannis, 2004). Model diagnostics were carried out to assess the fitness of the model to the data. The estimated return levels for different return periods revealed an increase in the value over years as this demands urgent attention and appropriate measure.

5 CONCLUSION

Extreme Value Distribution is a very powerful statistical technique for describing the unusual phenomena such as floods, wind guests, air pollution, earthquakes, hurricanes risk management, insurance and financial losses as rare events are difficult to quantify because they seem to follow no rules. Concerns over the environment cannot be over emphasized because whatever we do on earth has implication on the environment and poses hazard to human existence. Extreme Value Distribution plays a major role in monitoring and assessing this extreme event so as to be able to take appropriate measures towards its effect thus, the distributions were used to understand the extreme values in hydrology as this could help government and stakeholders in making policies regarding environmental and climatic issues.

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